APPROXIMATING THE WEIGHTING FUNCTION

TO BE USED IN

THE SMOS L2 PROCESSOR

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	Role	***Name***	***Date and signature***
Written by :	project scientist	Philippe Waldteufel SA Sonia Zine SA	
Approved by**:**	CNES Project Manager	François Bermudo	
Approved by :	Project manager	Achim Hahne	

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REFERENCES

1	SMOS Proposal (COP16)		Nov 1998
2	Mission Requirement Definition	5.0	Mar 2001
3	S0-TN-CBSA-GS-0001	1.b	Mar 2003
4	SO-TN-CBSA-GS-0006	2.a	Oct 2004

ACRONYMS

ASC	Ascending (pass)
CESBIO	Centre d’Etudes Spatiales de la BIOsphère
DESC	Descending (pass)
ECMWF	European Centre for Medium-range Weather Forecasting
ESA	European Space Agency
LST	Land Surface Temperature
NWC	National Weather Centres
PSU	Practical Salinity Unit
OS	Ocean salinity
Req	Required
SA	Service d’Aéronomie
SAG	Science advisory Group
SM	Soil Moisture
SMOS	Soil Moisture and Ocean Salinity Mission
SRD	System Requirement Document
SSS	Sea Surface Salinity
SST	Sea Surface Temperature
TB	Brightness temperature
TBC	To be confirmed
TBD	To Be Determined
TOA	Top Of Atmosphere
TX, TY	Polarised brightness temperature at antenna level and in antenna ref. frame
WS	Wind Speed
WSC	Wind Scatterometer

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1. CONTEXT AND PURPOSE

When processing SMOS data in order to retrieve Soil Moisture (SM) or Ocean Salinity (OS), it is necessary to make the best use of a priori information concerning the surface. In the cases of strongly inhomogeneous SMOS pixels, some contributions to the brightness temperatures (BT) should be assumed, while the retrieval is to be carried out only from the contribution to BT which corresponds to a suitable part of the scene.

Therefore we want to know, for every relevant incidence angle, the **fractions** of the SMOS pixel which correspond to vastly different cover types such as open water or forest. However, rather than fractional areas, we are talking here about weighted fractions.

The basis for computing the 2-D weighting function **WEF** to be applied to element areas in order to obtain weighted fractions is the synthetic antenna pattern of the MIRAS interferometer, This is equivalent to the **apodization function** applied to measured visibilities, and is also termed **equivalent array factor AF**.

To a first approximation, the AF is a rather narrow, centro symmetric function, independent of the location of the viewing point in the field of view, time independent 2-D pattern. Strictly speaking however, the AF exhibits **side lobes** (positive or negative) which extend far away from its axis; it is **not** centro symmetric, due to the star shaped baseline domain of the interferometer; it depends on the fringe wash (**FWF**) factor, which in turns varies with the location of the viewing point and possibly depends on the physical state of the instrument. Finally, the features of the AF will be modified in case any subsystem fails during the flight.

Furthermore, several additional steps are necessary in order to obtain the WEF from the AF.

In case exact AF and WEF computations are required, this may result in a rather heavy task in terms of computing time and/or data transfer.

This note reports simulations aiming at determine whether simplifying approximations to the apodization function or array factor AF can be used when computing weighting functions in the SMOS L2 processor, and assessing the impact of simplifying further the WEF computation.

This analysis is mainly necessary to SM processing, since the mixed pixels are an important part of the land surface scenes. However the results will also apply to ocean scenes. Furthermore, the AF must be considered for both cases, in order to compute accurately the radiative component due to the sky.

2. TRUNCATING THE ARRAY FACTOR AF

AF patterns without FWF factor computed by DEIMOS are shown on Figure 1 and (zoomed) Figure 2The patterns are normalized to 1 maximum value; the central part of the main lobe is saturated.

Results of SMRS study (SMRS study final report, SMOS-FR-ACR-SA-007, section 2.2.3) have shown that truncating the AF to its main lobe results in negligible errors on retrieved SM. In the SMRS study, the truncation level was selected to be 0.05 (contour shown on Figure 2)

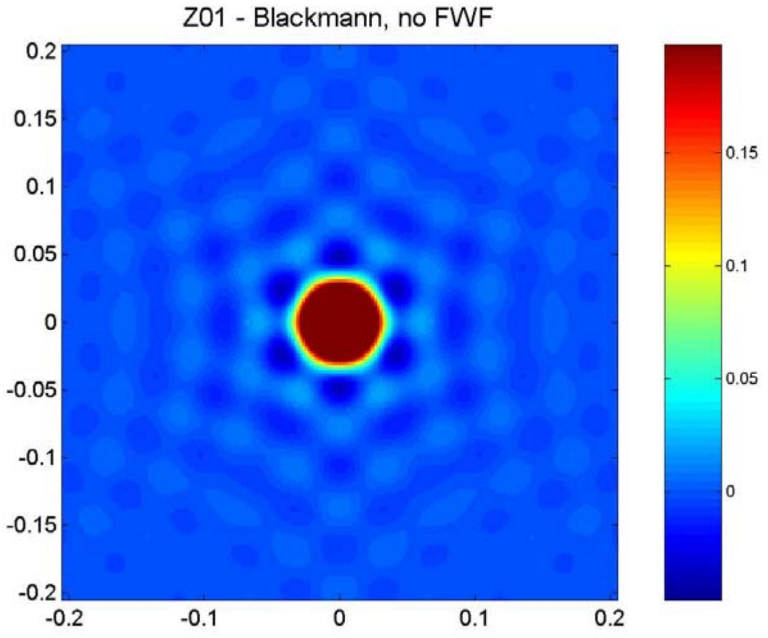

Figure 1

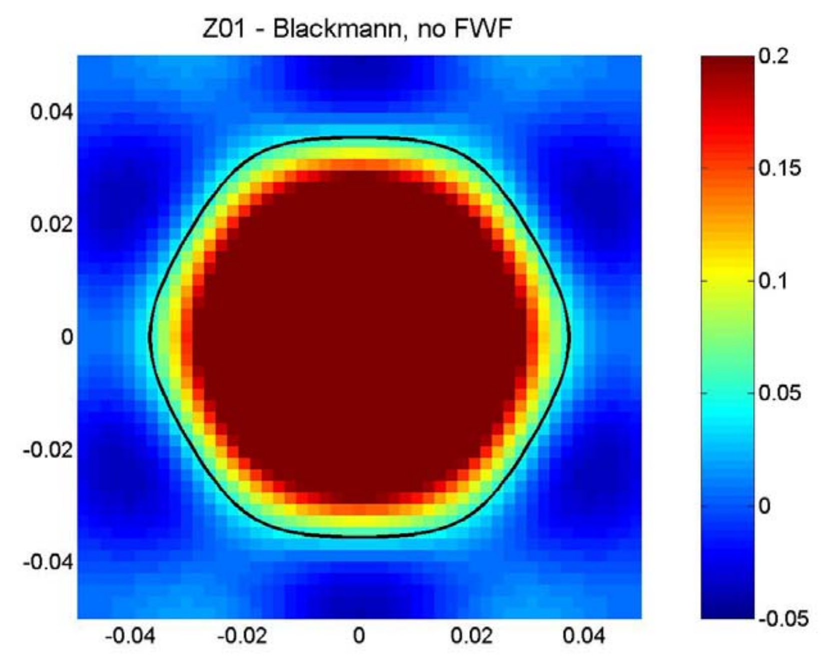
Z01 - Blackmann, no FWF
0.04
0.02
0
-0.02
-0.04
-0.04
-0.02
0
0.02
0.04
0.2
0.15
0.1
0.05
0
-0.05

Figure 2

3. APODISATION FUNCTION SIMULATIONS

The expression of the Array factor is as follows (from: Computation of the Synthetic Antenna Directional Gain as interface to L2 Processor, Ref: Synthetic Antenna Di#4B39A1, Deimos/ESA 26 Apr 2005, 3pp):

$$AF_{eq}\left(\xi,\xi',\eta,\eta'\right)=\frac{\sqrt{3}}{2}d^2\sum_m\sum_n W\left(u_{mn},v_{mn}\right)\cdot\tilde{r}\left(-\frac{u_{mn}\cdot\xi+v_{mn}\cdot\eta}{f_0}\right)\cdot e^{j2\pi\left(u_{mn}\left(\xi-\xi'\right)+v_{mn}\left(\eta-\eta'\right)\right)}$$

where

- W is the apodisation function
- r is the fringe-washing function which accounts for the spatial decorrelation between antennas.
- u,v are the baseline coordinates in the frequency domain
- d is the antenna element spacing (= 0.875)
- $f0$ is the central frequency (1413 MHz)
- ξ, η are the central director cosines (DC) coordinates; ξ', η' are running DC coordinates.

Simulations were performed by Deimos, using the Blackmann exact window. Several cases were considered, depending whether the fringe wash factor is included or not, and whether some LICEF failures are assumed.

For including the fringe wash (FWF) factor, the selected (ξ,η) coordinates are (ξ =0.32, η =0.25). The bandwidth to central frequency ratio BW/f_0 (which has to be introduced in the FWF factor) was taken equal to 2% (remember that for SMOS it is slightly smaller, as the equivalent bandwidth is 19 MHz rather than 27 MHz).

Selected cases are summarized on Table 1.

Table 1

File name	FWF factor	LICEF Failures
T01	No	None
T02	Yes	None
T21	No	A01
T22	Yes	A01
T31	No	A21
T32	Yes	A21
T41	No	A21 & B11
T42	Yes	A21 & B11

The map of the references for LICEFs is shown on the next **figure**.

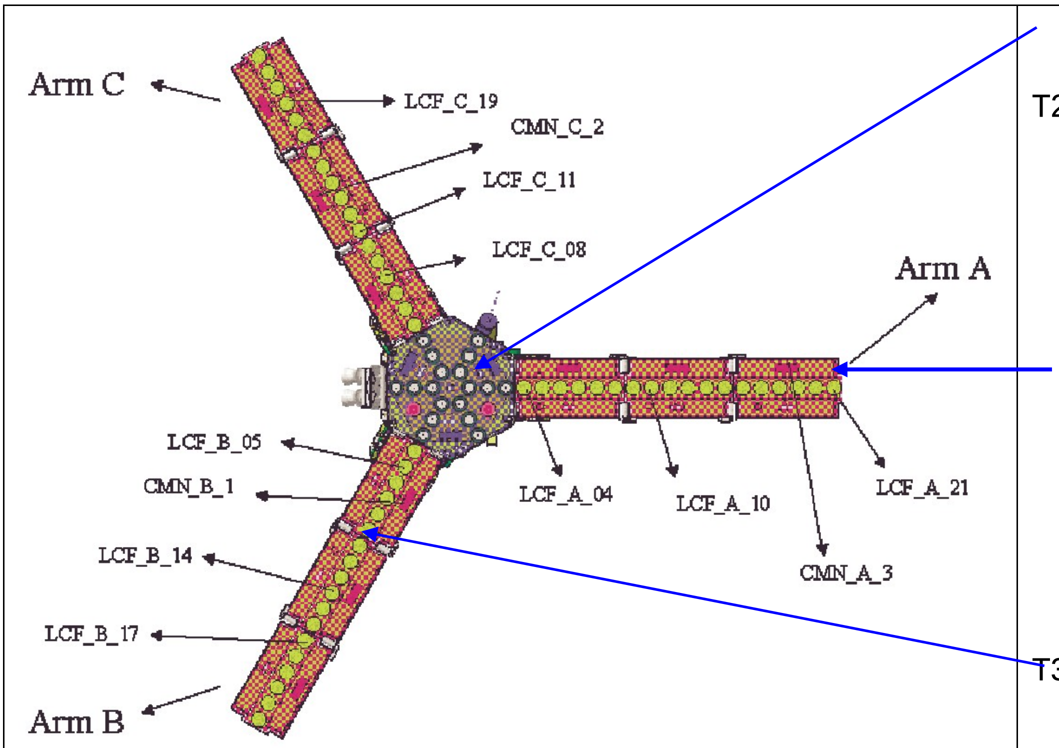

Figure 3

From CASA S20100-TNO-002-E1-R1_LICEFS note, figure 10.3

4. CENTROSYMETRIC APPROXIMATION

The following analytical approximation has been tuned to the AF with no FWF effect:

Fxy ≈ (sin(kf*(xr+1e-10)) ./ (kf*(xr+1e-10))).^kk ./ (1+kg*(xr).^kh) **Eq 1**

where **xr** is the distance in the DC coordinates; negative Fxy values are set to zero.

Best fit parameters are shown on Table 2; RMS=0.00612

Table 2

	kf	kg	kh	kk
initial	75.00	800.0	2.0000	1.2000
Final	73.30	524.5	2.1030	1.4936

Figure 4 and Figure 5 illustrate the quality of the fit over the main AF lobe. As indicated above, the RMS difference is about 0.6% (with maximum value of AF normalized to 1).

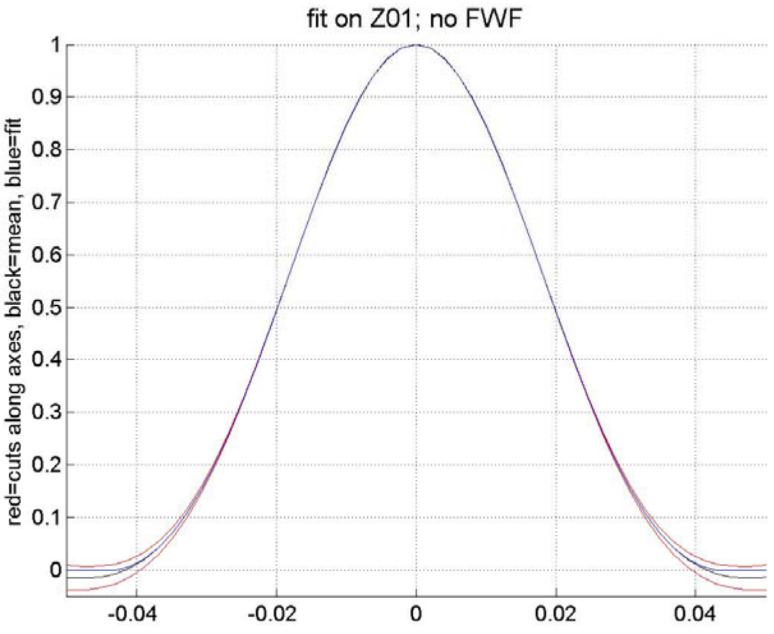

Figure 4

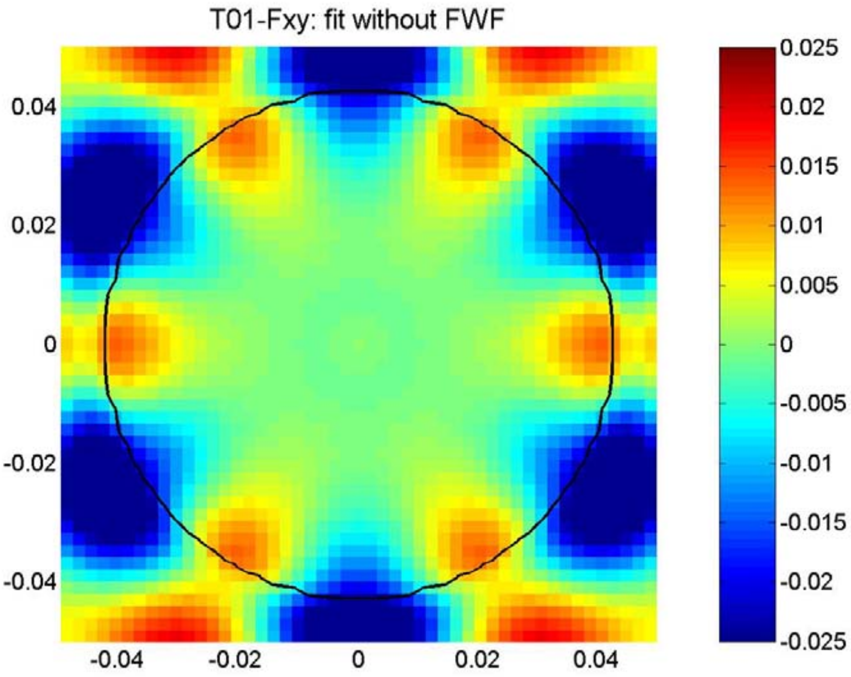
T01-Fxy: fit without FWF
0.04
0.02
0
-0.02
-0.04
-0.04
-0.02
0
0.02
0.04
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025

Figure 5

5. EFFECTS OF FRINGE WASH FACTOR AND LICEF FAILURES

Figure 6 shows the impact of the FWF factor. It is a very slight flattening and elongation along an axis close to the main diagonal, i.e. actually the radial direction in the DC coordinates, remembering that the computation is carried out for (ξ =0.32, η =0.25).

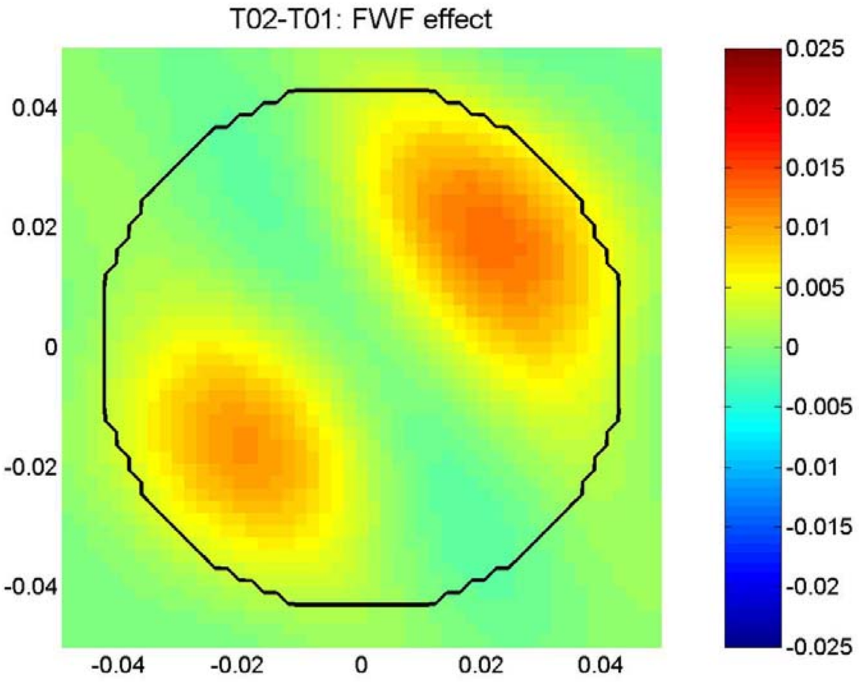

Figure 6

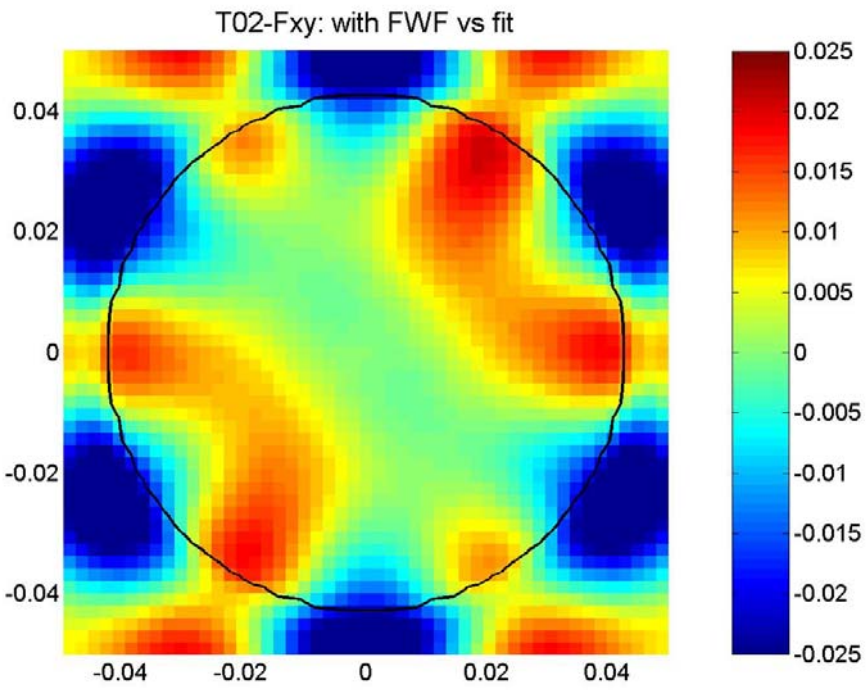

Figure 7

Figure 7 shows the comparison with the fit Fxy obtained above without FWF. The impact of FWF is obvious, but hardly ever exceeds 2%.

Figure 8 and Figure 9 shows the impact on AF of examples of single (Figure 8) and double (Figure 9) LICEF failures, chosen as depicted on Figure 3. Figure 10 and Figure 11 illustrate comparisons with the function Fxy fitted to the (no failure, no FWF) pattern.

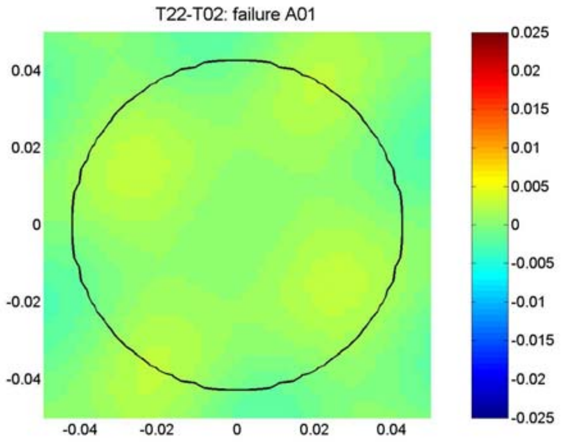

Figure 8

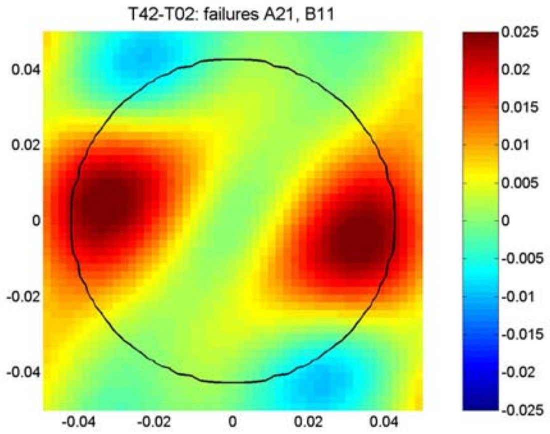

Figure 9

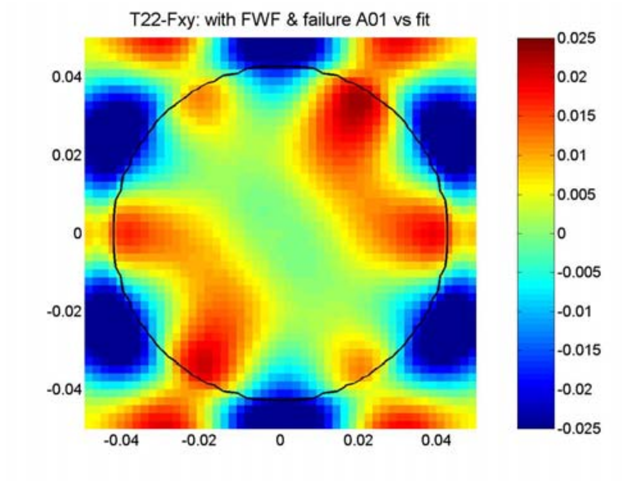

Figure 10

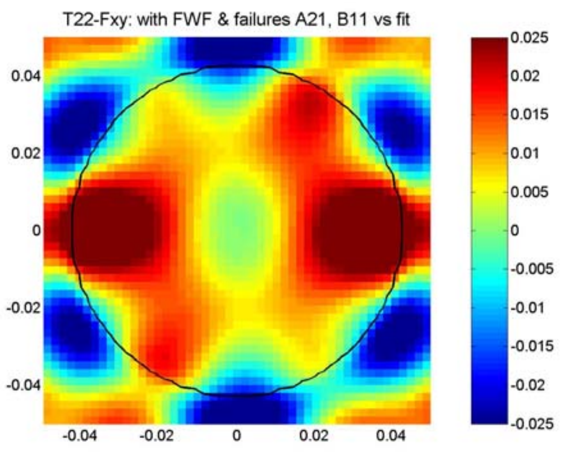

Figure 11

The Table 3 below shows the quality of the results as well as pattern differences for various cases, expressed through the RMS value (this is equivalent to a chi square, since several hundred pattern data are used for the fit while only 4 parameters are fitted). Fxy is the approximation

All functions are normalized to maximum unity value. RMS is computed over the main lobe

Table 3

Fitting error	**RMS value**		**Failure effect (no FWF)**	**RMS value**
T01-Fxy	0.000691		T21-T01	0.001021
T02-Fxy	0.003429		T31-T01	0.002432
T21-Fxy	0.000330		T41-T01	0.008143
T22-Fxy	0.004449			
T31-Fxy	0.001741		**Failure effect (with FWF)**	
T32-Fxy	0.005915		T22-T02	0.001019
T41-Fxy	0.007452		T32-T02	0.002486
T42-Fxy	0.011634		T42-T02	0.008205
FWF effect				
T02-T01	0.004120			
T22-T21	0.004118			
T32-T31	0.004174			
T42-T41	0.004182			

6. SIMULATIONS FOR SM RETRIEVAL AND RESULTS

We must first account for fringe wash (FWF) effect. As seen on Figure 7, the main effect of the fringe wash factor is an elongation along the radial direction away from boresight.

From Bara et al. (Bara J., Camps A., Torres F and Corbella I.: Angular resolution of two-dimensional, hexagonally sampled interferometric radiometers, Radio Science, vol 33, pp 1459-1473, Sept. 1998), the following approximate expression is proposed:

$$\frac{\Delta\alpha^{f}_{-3\mathrm{dB}}}{\Delta\alpha^{0}_{-3\mathrm{dB}}} = \sqrt{1+\frac{\mathrm{W}^2}{3}\,\frac{\tan^2(\theta)}{(\Delta\alpha^{0}_{-3\mathrm{dB}})^2}} \qquad \textbf{Eq 2}$$

Where

- $\Delta\alpha^{f}_{-3\mathrm{dB}}$ is the 3 dB width of the synthetic gain pattern when accounting for the FWF effect;
- $\Delta\alpha^{0}_{-3\mathrm{dB}}$ is the 3 dB width of the synthetic gain pattern in absence of FWF effect;
- W = BW/f0;
- θ is the angle with respect to antenna boresight.

For the location selected in AF simulations, θ is close to 24°. Over the SMOS extended alias free field of view, θ never reaches 50°.

For the Blackmann apodization function, the maximum broadening factor due to FWF is then found close to 1.023. This is consistent with numerical values computed by Bara et al (for a larger MIRAS configuration) and shown in their table 2.

In the SMRS software, the direct simulation is carried out with the full apodization window over a ± 0.15 (ξ,η) domain, ignoring the fringe wash factor. In the retrieval, we test together, against an "exact" retrieval using an identical AF:

A **truncation** limited to the central to whole main lobe, depending on the threshold **th** (in director cosine units); to be compared with curves on Figure 4.

The **analytical centrosymmetric** approximation presented above

The **fringe wash** effect, which must then be simulated in an **opposite** way (i.e. using a "-" sign in the square root of formula (2) in order to produce a narrowing of the AF along the radial).

Finally, the fringe wash effect (corrective term in the formula) is **scaled** by a factor c=1, 3 or 10, in order to detect how the retrieval performances deteriorate as the approximation for the AF worsens.

Figure 12, built for a scaling factor c=3, shows that the magnitude of the differences with respect to the basic fit Fxy (computed with no fringe wash) is at least comparable to the impact of the LICEF failures tested above. This figure, incidentally, shows that elongating radially the AF pattern through use of formula (2) does distort the AF pattern in a way similar to what is yielded by numerical computation (see Figure 6)

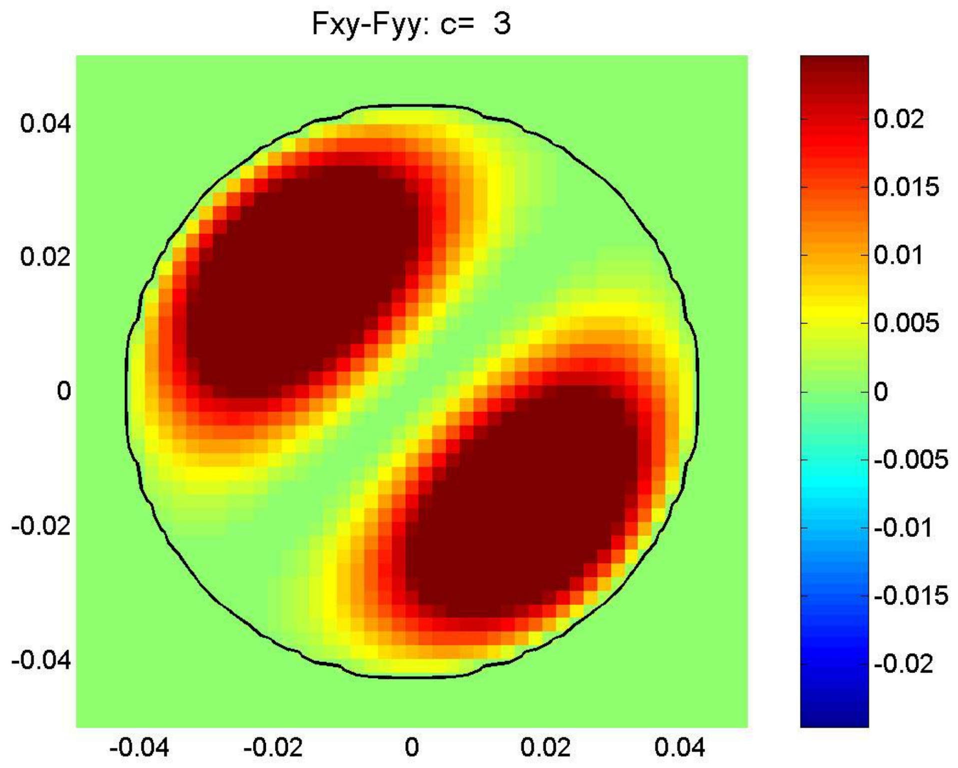

Figure 12

The retrieval is carried out on a Western Europe simulated scene (see Figure 13).

The fit formula used in the simulation was actually very slightly different from equation (1) because, in the SRS software, apodization functions where computed with 3 elements on the rear side of each arm.

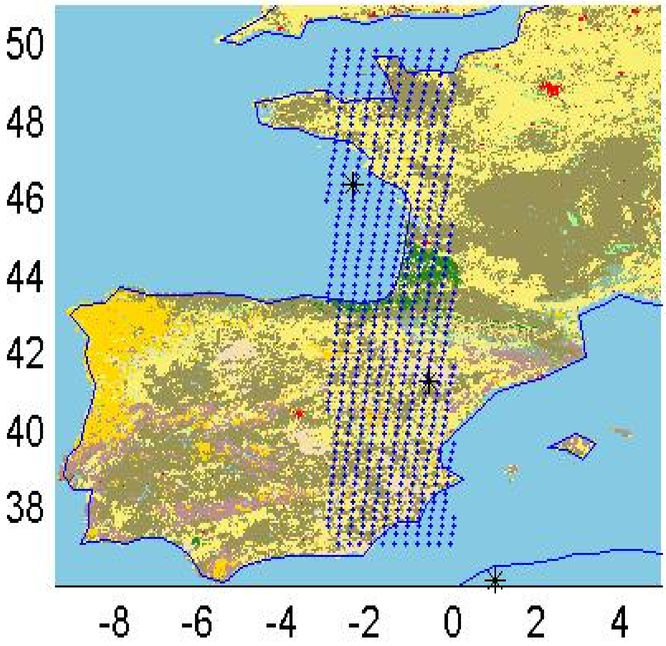

Figure 13

In the following Table 4, biases and standard deviations for soil moisture (expressed as fraction) are given in columns F to H and sorted into 4 categories (col. D), depending on the amount of open water present in the pixels. Other inhomogeneities (forested areas, for example) are also present; hence the last line in each test is also representative of inhomogeneous pixels. The first line in each test has little meaning because the quality of the SM retrieval is very poor (see column I).

Table 4

A	B	C	D	E	F	G	H	I
#	Options		land fraction (%)	N	mean (ex.-ap.)	std wrt mean (ex.-ap.)	std wrt 0 (ex.-ap.)	std th appro
1	step 16 km		0 to 40	135	**0.0546**	**0.1636**	**0.1725**	0.0965
	th = 0.0392	c=1	40 to 75	81	**0.0019**	**0.0079**	**0.0081**	0.0276
	smearing	yes	75 to 95	92	**0.0045**	**0.0078**	**0.0090**	0.0234
	Ds	yes	95 to 100	858	**0.0015**	**0.0051**	**0.0053**	0.0210
2	step 16 km		0 to 40	135	-0.0186	0.0740	0.0763	0.0767
	th = 0.0340	c=1	40 to 75	81	0.0042	0.0122	0.0129	0.0279
	smearing	yes	75 to 95	92	0.0091	0.0085	0.0125	0.0237
	Ds	yes	95 to 100	858	0.0016	0.0052	0.0054	0.0210
3	step 24 km		0 to 40	59	-0.1043	0.1521	0.1844	0.1110
	th = 0.0290	c=1	40 to 75	34	-0.0002	0.0358	0.0358	0.0267
	smearing	yes	75 to 95	41	0.0229	0.0176	0.0289	0.0248
	Ds	yes	95 to 100	384	0.0018	0.0074	0.0076	0.0210
4	step 24 km		0 to 40	59	0.0846	0.1816	0.2003	0.1063
	th = 0.0392	c=3	40 to 75	34	0.0005	0.0089	0.0089	0.0274
	smearing	yes	75 to 95	41	0.0027	0.0090	0.0094	0.0235
	Ds	yes	95 to 100	384	0.0014	0.0051	0.0053	0.0209
5	step 24 km		0 to 40	59	0.1164	0.2119	0.2418	0.1157
	th = 0.0392	c=10	40 to 75	34	-0.0003	0.0136	0.0136	0.0273
	smearing	yes	75 to 95	41	-0.0070	0.0151	0.0166	0.0226
	Ds	yes	95 to 100	384	0.0009	0.0058	0.0059	0.0208

Performance on retrieved SM (in %);

Ex = exact AF(= reference); ap. = approximated AF

The most significant result (col. H) shows that for the "nominal" AF approximation, the resulting error on SM is around 0.5% with little open water, and around 1% for significant fractions of open water.

For cases 2 and 3 (higher truncation threshold selected for the fitted AF), the differences become higher. Definitely, it seems better to keep the whole main lobe of the apodization function.

For case 4 (simulating a distortion comparable to those resulting from LICEF failures), the error is not changed. It would become higher however if the distortion was larger (case 5).

7. COMPUTING THE ARRAY FACTOR

Using the approximation proposed above, the steps to compute AF are as follows:

7.1 LOCATE FINE GRID ZONE AROUND EACH NODE OF THE SMOS GRID

The size if the fine grid area is determined by the SMOS requirements, which stipulate 50 km as maximum equivalent diameter of the SMOS pixel (i.e. half power contour of the WEF) and 1.5 as maximum value of the elongation factor. This gives 61 km as maximum size. Since for the Blackmann function the main lobe is very nearly twice as wide as the half power contour, the resulting extent of the fine grid zone is **122 km**.

If there is a constraint on this figure, methods can certainly be found to (i) limit the number of cases where the weighting function must be computed, and (ii) limit the size of the required zone, as the SMOS pixel is actually almost always smaller than its maximum allowed size.

7.2 COMPUTE DIRECTOR COSINES COORDINATES FOR EACH NODE IN THE FINE GRID ZONE

Assume the concerned SMOS grid node is characterized by its geographical angular coordinates, i.e. viewing angle θ_{G0} and azimuth ϕ_{G0}.

Then the geographical knowledge of the fine grid zone allows computing angular coordinates θ_G and ϕ_G. for each node of the fine grid array.

From these angles, the director cosines are readily obtained (Matlab © code notation):

```
ct=cos(tilr); st=sin(tilr);
ci=cos(incr);  si=sin(incr);
xm=sin(teg)*cos(phg);
ym=sin(teg)*sin(phg);
zm=cos(teg);
vax=ci*xm+si*ym;
vay=ct*(-si*xm+ci*ym)-st*zm;
vaz=st*(-si*xm+ci*ym)+ct*zm;
phb=atan2(vay,vax);
ra=sqrt(vax.^2+vay.^2);
teb=atan2(ra,vaz);
xhi=sin(teb).*cos(phb);
eta=sin(teb).*sin(phb);
```

Here:

tilr is the tilting angle of the antenna plane wrt to local horizontal plane (radians)
incr is the angle of the (XOY) plane (PROTEUS notations; this is very close to the instantaneous orbit plane, but for yaw) wrt the meridian plane (radians)
teg, phg are arrays for θ_G and ϕ_G.

teb, phb are arrays for computed θ and ϕ in the antenna frame;
xhi and eta are arrays for director cosines.

7.3 COMPUTE AF FOR THESE NODES OF THE FINE GRID

This is done using the approximated AF function.

8. FROM AF TO WEIGHTING FUNCTION WEF

In the exact simulations, 2 further steps are needed :
Account for the "smearing effect" due to integration along the track;
Account for the variation of the integrating element with incidence angle, as the fine grid area does not lie on a plane.
Table 5 shows the results of
neglecting the smearing effect, assuming the integration time corresponds to 8 km along the track in the direct simulation (test #6)
ignoring the variation of incidence angle, i.e. assuming in the retrieval that the Earth surface is a plane around the SMOS grid node (test #7)
No significant impacts appears when comparing the errors to those obtained for test #1.

Table 5

A	B	C	D	E	F	G	H	I
#	Options		land fraction (%)	N	mean (ex.-ap.)	std wrt mean (ex.-ap.)	std wrt 0 (ex.-ap.)	std th appro
1	step 16 km		0 to 40	135	**0.0546**	**0.1636**	**0.1725**	0.0965
	th = 0.0392	c=1	40 to 75	81	**0.0019**	**0.0079**	**0.0081**	0.0276
	smearing	yes	75 to 95	92	**0.0045**	**0.0078**	**0.0090**	0.0234
	Ds	yes	95 to 100	858	**0.0015**	**0.0051**	**0.0053**	0.0210
6	step 24 km		0 to 40	59	0.0580	0.1692	0.1789	0.0998
	th = 0.0392	c=1	40 to 75	34	0.0014	0.0099	0.0100	0.0275
	smearing	no	75 to 95	41	0.0083	0.0066	0.0106	0.0240
	Ds	yes	95 to 100	384	0.0016	0.0050	0.0052	0.0209
7	step 24 km		0 to 40	59	0.0783	0.1730	0.1899	0.1006
	th = 0.0392	c=1	40 to 75	34	0.0020	0.0083	0.0085	0.0275
	smearing	yes	75 to 95	41	0.0060	0.0077	0.0098	0.0238
	Ds	no	95 to 100	384	0.0015	0.0050	0.0052	0.0209

9. CONCLUSIONS

It is shown on a representative sample of simulated data that fitting the apodisation function by a centrosymmetric simple approximation and neglecting the FWF factor yields errors of the order of 0.5% on the retrieved soil moisture for SMOS pixels with weak open water contamination; the errors climb to about 1% when the open water fraction becomes significant. The best thresholding choice is to retain the whole main lobe of the AF.

No significant effect of the approximation is detected when the impact of 1 or 2 LICEF failures is simulated.

This exercise has been done for the exact Blackmann function. A different fitting function should be computed in case of selecting another apodization window.

Furthermore, it is found that no additional significant error result when the apodization function is directly taken as the weighting function, ignoring both smearing and Earth sphericity effects.

These errors are about one order of magnitude below the SMOS SM requirement for pixels with small open water contamination. Although they are higher when significant fractions of open water are present, they are believed to be acceptable.

10. ACKNOWLEDGMENTS

We are grateful to the DEIMOS team for computing the apodization functions used in this note.

11. REFERENCES

SMRS study final report, SMOS-FR-ACR-SA-007, section 2.2.3

Computation of the Synthetic Antenna Directional Gain as interface to L2 Processor; Ref: Synthetic Antenna Di#4B39A1, Deimos/ESA 26 Apr 2005, 3pp

Bara J., Camps A., Torres F and Corbella I. : Angular resolution of two-dimensional, hexagonally sampled interferometric radiometers, Radio Science, vol 33, pp 1459-1473, Sept. 1998